Public Policy and Competitive Runs

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Abstract

In competitive industries, foreseeable policy changes lead to inevitable runs which increase the volatility of investment. We show that this phenomenon, well-known in the case of production caps, also applies to taxes, and occurs whether policy changes apply to new entrants only or equally to all firms. Looking at the case of raising taxes (or removing a subsidy) we find that the size of runs increases with the magnitude of the tax, and that runs are smaller when the policy change affects all firms. Runs accelerate investment compared with the socially optimal path, lowering welfare, and this effect can be quantitatively important. These results have implications for a broad range of policies, such as the removal of renewable energy subsidies in some European countries.

Key words: Competition, Investment, Uncertainty, Welfare

J.E.L. codes: (C61, D41, D62)

1. Introduction

When in the summer of 2020 the French government announced a hundred-billion euro stimulus plan, France Relance, to alleviate adverse consequences of the COVID-19 pandemic on economic activity, this was with the understanding that measures in the package, such as significant tax breaks for firms, would extend over a two-year period, a delay presumably reflecting the expected time needed for the economy to recover. As much therefore as policy interventions like this one can arise unexpectedly (ahead of the lockdowns measures which sharply curtailed economic activity, the pandemic was hardly anticipated), there are also circumstances where the timing of policies is known to some extent by economic actors. Such knowledge can arise because policy timing is explicit, or because it is closely tied to events whose exact timing, though unknowable in advance, is nevertheless predictable, like economic recovery from a shock, or the moment that environmental parameters reach a critical value. For an example of the latter sort, consider another important policy objective, decarbonization, whose achievement rests in part upon promoting green investment. While the road ahead is still long for many countries, in the fall of 2020, Sweden and Norway jointly announced that their green power support schemes would be phased out by 2035. Moreover, these schemes would be closed to new participants as of the beginning of 2022, because both countries deemed that their renewable energy production targets had been largely met.

It is often preferable that economic actors anticipate policies, for a variety of reasons. For businesses for example, this can help firms ease their technological or organizational transitions so as to economize on adjustment costs. In addition, uncertainty about the timing of policies creates a spurious option value of waiting which delays investment (Dixit and Pindyck 1994, Hassett and Metcalf 1999). If the implementation of a policy can be anticipated however, as in the preceding examples, then in competitive industries this anticipation has less familiar, and less desirable consequences. This is because the anticipation of a policy intervention can drive investment in competitive industries to overreact in equilibrium, and significantly raise its variability. The reason is that anticipated changes create the conditions for competitive runs, a phenomenon which has previously been studied in the context of very specific measures like quotas and caps on investments, but which we argue in this paper arises under a much broader range of policy measures and which we show is likely to be quantitatively significant.

The concept of a run in Economics has historically been most closely associated with movements in financial asset values. In the Diamond and Dybvig (1983) model of bank runs, rather than resulting from a gradual, dynamic process, the run arises as one of several equilibria, driven by individual beliefs. In Krugman's (1979) balance of payment crisis model, a run occurs in the face of dwindling reserves before these are depleted, as individuals protect their assets, but the framework this result is derived in is a Keynesian model without optimization. A common feature to both these classic studies is that the run springs from the knowledge that a certain stock is going to be fully exhausted, be it liquid bank reserves in Diamond and Dybvig's model, or the central bank's foreign currency reserves in Krugman's.

Later studies have focused on runs that occur in a market where profits are stochastic and investment is irreversible. In these studies, the runs emerge in equilibrium when there is a cap on aggregate investment. As the endpoint of an investment opportunity nears, firms cease to wait for an optimal threshold to be reached and a mass of investment occurs. Applications involving this dynamic pattern of a run arise naturally in a variety of areas, ranging from caps on foreign investment (Bartolini 1995), to immigration quotas (Moretto and Vergali 2010), or policies restricting land use (Di Corato, Moretto and Vergalli 2013). Some of these articles go a step further by incorporating an adverse externality which motivates imposing the cap and studying its optimal timing.

In the present article, we study a model of competitive investment under uncertainty and describe the general conditions that can give rise to a run. These include, for example, anticipated changes in tax rates and more broadly any measure that generates a kink in the revenue or cost functions that firms face, and thereby cause the investment threshold function to jump. Like a cap (which can be interpreted as an infinite tax) therefore, a tax on entries once a market size or industry capacity threshold is met results in a run. The threshold at which the run starts is inversely related to the size of the tax.

Our analysis shows that such a run occurs because the worsening of conditions after a policy intervention (tax or subsidy withdrawal) slows the subsequent entry process and the drop in prices that each entry would bring, thereby improving the revenue flow that existing firms enjoy. As existing firms would then make supranormal profit, a mass of new firms is attracted into the market generating the run. This process ultimately eliminates the possibility that the improved future revenue process might raise firm value above the normal return. More specifically, the run brings about an immediate drop in price which balances the improved future revenue process.

To verify this intuition, we also study an alternative policy, a tax on flow or operating cost rather than fixed cost, which affects active and inactive firms equally. In this situation all firms have the same revenue flows after the policy intervention yet sill a run emerges, confirming that the same reasoning applies: it is not any specific future tax advantage but rather the slowing of future entries and resulting rise in revenues which drives investment decisions. This feature springs from the endogenous nature of the revenue process – it is based on firms' entries and those occur when their net value is zero, due to free entry. Thus, the run occurs not because it rewards its winners with an advantage compared to those who were not lucky enough to enter before the policy change. On the contrary, the run is a dynamic equilibrium pattern that keeps value equal for all – at the normal return.

Finally, we turn to the welfare properties of competitive runs. In the present paper, we consider an optimal policy where the regulator chooses a tax that exactly reflects the investment externality, implying that at any moment firms face the correct economic cost. For such a policy, we find that the run reduces welfare by excessively accelerating investment, which destroys a valuable option to wait.

From a policy perspective, our analysis relates directly to the studies of policy uncertainty and investment referred to above. A current application of this literature is investment in renewable energy generation. The effect of incentivizing policies in this area has both studied and estimated extensively (Linnerud, Andersson and Fleten 2014). In such studies, policy effects generally are estimated for small projects where individual investors are price-takers in the electricity market and both the level of policy and the electricity price fluctuate stochastically. Different policies which have been considered include lump-sum investment subsidies and feed-in tariffs which shield firms from lower electricity prices. More recently, this literature has begun to highlight the risk associated with the withdrawal of subsidies (Nagy, Hagspiel and Kort 2021), as highlighted in our opening paragraph. We contribute to this discussion by complementing existing models through the introduction of competitive equilibrium and the run behavior this generates, both in the case of lump-sum subsidies and feed-in tariffs.

Section 2 presents the assumptions underlying our model. Section 3 studies competitive equilibrium in the absence of policy intervention. Section 4 considers a tax on investment and derives the resulting behavior, characterizing the effect of parameters on the magnitude and timing of the competitive run. In Section 5, we extend this analysis to the case of a tax on operating costs and compare the two forms of policy. Section 6 studies welfare and provides a numerical illustration of our results.

2. The model

A continuously evolving demand is served by a competitive industry. At any time $t \in R_+$, the price is

(1) $$P_t = X_t \cdot f(Q_t),$$

where X_t is an exogenous shock, Q_t is industry capacity, and $f(Q)$ is a differentiable downward-sloping function with $\lim_{Q \to \infty} f(Q) = 0$. The exogenous shock follows a geometric Brownian motion

(2) $$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t$$

where μ is the drift parameter, $\sigma > 0$ the volatility parameter, and dZ_t is the increment of a standard Wiener process, uncorrelated across time and satisfying $E(dZ_t) = 0$ and $E[(dZ_t)^2] = dt$ at any t.

The industry consists of identical risk-neutral firms, of which a mass $Q > 0$ is active initially. Active firms own a unit of capacity, produce up to their capacity, and have a unit operating cost w. Inactive firms become active by acquiring a unit of capacity at cost K. Once active, firms cannot suspend operations or exit to become inactive again. All firms have the same constant discount rate r. We suppose $r > \mu$ to focus on the case where firm value is finite.

When industry capacity reaches a predetermined trigger level, a policy intervention permanently alters the industry's cost parameters. The first policy we study is an increase in capacity cost from K to K'. Such an increase can result either from the introduction of a tax on investment at a constant rate τ, in which case $K' = (1+\tau)\cdot K$, or from the withdrawal of a lump-sum investment subsidy S, in which case $K' = K + S$. We also study changes in operating cost which may apply to either all firms or only to new entrants, and finally decreases in cost.

3. Industry equilibrium without policy intervention

To lay the groundwork for our analysis we begin with the industry equilibrium if K and w remain at their initial levels forever. Firms therefore face the same situation as investors in Leahy (1993), whose analysis we use to present the optimal investment policy and competitive equilibrium.

The decision to enter the market is driven by expected profitability, and therefore takes place only when X_t is sufficiently large. In particular, given the current market quantity, Q, a firm enters the market only if X_t hits an entry threshold which we denote by $X^*(Q)$. To find this threshold, let $V(Q,X)$ denote the value of an active firm (we drop the time subscript on Q and X for notational convenience).[1] A no-arbitrage argument (see Appendix A) shows that $V(Q,X)$ is a continuous and differentiable function of X,

$$V(Q,X) = Y(Q)\cdot X^{\beta} + \frac{X\cdot f(Q)}{r-\mu} - \frac{w}{r}, \tag{3}$$

where $Y(Q)$ is described further below and β is the upper root of

[1] For simplicity of exposition, we also suppose throughout that the initial levels of Q and X are sufficiently small so that any run occurs in the future.

(4) $$\frac{1}{2}\cdot\sigma^2\cdot x^2+\left(\mu-\frac{1}{2}\cdot\sigma^2\right)\cdot x-r=0.$$

In Appendix A we also show that $\beta > 1$. In addition, we show there that:

(5) $$E_{X_0=X}\left[\int_0^\infty (X_t\cdot f(Q)-w)\cdot e^{-r\cdot t}dt\right]=\frac{X\cdot f(Q)}{r-\mu}-\frac{w}{r}.$$

From Eq. (5) it follows that the second and third terms in Eq. (3) represent the expected present value of the profit flow the firm would obtain if Q at its current level forever. This implies that the first term in Eq. (3), namely $Y(Q)\cdot X^{\beta}$, accounts for the effect of future changes in industry profitability on the value of an active firm. $Y(Q)$ is found together with the entry threshold $X^*(Q)$ via two boundary conditions which apply at any moment at which investment occurs. The first is the *Value Matching* condition,

(6) $$V(Q,X^*(Q))=K.$$

Eq. (6) states that the net value of entry (i.e., of becoming an active firm) is zero. The Value Matching condition is due to instantaneous competition between inactive firms. It holds for any investment threshold, and not merely for the optimal threshold $X^*(Q)$.

The second condition which applies when investment occurs is the *Smooth Pasting* condition,

(7) $$V_X(Q, X^*(Q)) = 0.$$

Eq. (7) is an optimality condition, which requires the values of active and inactive firms to have the same slope with respect to X at the instant of becoming active.

Substituting the specification of Eq. (3) into the Value Matching and Smooth Pasting conditions determines a unique investment threshold

(8) $$X^*(Q) = \hat{\beta} \cdot (r - \mu) \cdot \frac{K + \frac{w}{r}}{f(Q)},$$

where $\hat{\beta} = \frac{\beta}{\beta - 1} > 1$. The uncertainty wedge $\hat{\beta}$ scales up the investment threshold compared with the net present value rule, to account for the presence of uncertainty and irreversibility (see Dixit and Pindyck, 1994, Ch. 5, Section 2). Note from Eq. (8) that $\frac{\partial X^*(Q)}{\partial \beta} < 0$ and note from Eq. (4) that $\frac{\partial \beta}{\partial \sigma} < 0$. This implies that the investment threshold increases with demand volatility. Finally, because $f(Q)$ is downward sloping, $X^*(Q)$ is an increasing function of industry capacity.

The threshold policy defined by Eq. (8) implies a path of industry capacity in (Q, X) space as represented in Figure 1. At a point like A that lies inside the region below $X^*(Q)$, small movements of the continuous demand shock X_t shift the industry's position vertically but do not provoke changes in industry capacity. As soon as X_t hits $X^*(Q_A)$ however, investment occurs, increasing industry capacity. This raises the investment threshold, so the industry lies again below $X^*(Q)$ and further investment is postponed until the next moment X_t hits the threshold function.

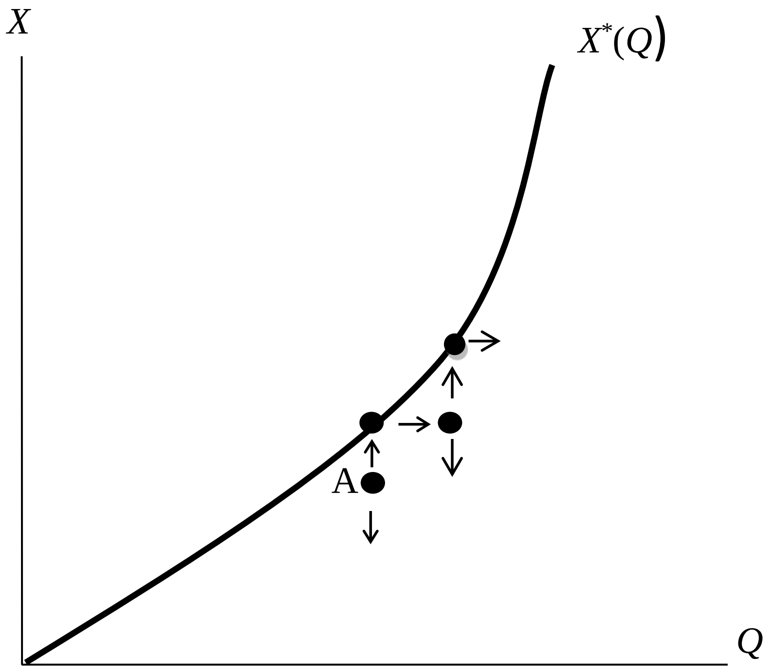

Figure 1. Entry process in the absence of policy

Leahy (1993) observes that this pattern of investment in a competitive industry results in a cap on the price process $P_t \leq P^*$, defined by

$$P^* = X^*(Q) \cdot f(Q) = \hat{\beta} \cdot (r - \mu) \cdot \left(K + \frac{w}{r}\right). \tag{9}$$

The term $Y(Q)$ in Eq. (3) therefore corrects the perpetual profit flow terms in the firm value expression to account for the truncation of the price process at P^* resulting from future entries. Substituting Eqs. (3) and (8) into either Eq. (6) or Eq. (7) gives that under the optimal policy $Y(Q)$ is given by

$$Y^*(Q) = -\frac{(\beta-1)^{\beta-1}}{\beta^{\beta}\cdot(r-\mu)^{\beta}\cdot\left(K+\frac{w}{r}\right)^{\beta-1}}\cdot\left(f(Q)\right)^{\beta} \quad (10)$$

The correction term $Y(Q)\cdot X^{\beta}$ depends on current profitability. Because demand is downward sloping, the higher the current industry capacity Q the smaller the correction term (in magnitude), and higher costs K or w have a similar effect.

4. Industry equilibrium with a policy that increases fixed cost

Having described the basic competitive equilibrium in the previous section, we now can incorporate a policy intervention into the analysis. The intervention we study in this section is an increase in unit capacity cost to $K' > K$. The increase in cost is triggered once industry capacity pass a predetermined level $\bar{Q}$. The policy and its timing are both known to firms.

In the case of a cap, Bartolini (1993) and Di Corato and Maoz (2019) show that the analysis of industry equilibrium starts by repeating the steps presented in the previous section without policy intervention, and parts ways only when the condition for optimal investment is introduced. That is, first a no arbitrage argument implies that the value of an active firm has the form in Eq. (3). Next, because of instantaneous competition between inactive firms, the *Value Matching* condition Eq. (6) must hold (with *K'* at the RHS, instead of *K*). However, the discontinuity in industry conditions at $\bar{Q}$ implies that the *Smooth Pasting* condition is replaced by a general optimality condition involving *Complementary Slackness*,[2]

[2] See Di Corato and Maoz (2019) for a proof of the validity of this condition.

(11) $$V_X\big(Q, X^*(Q)\big) \cdot \frac{dX^*(Q)}{dQ} = 0.$$

Eq. (11) holds if either $V_X\big(Q, X^*(Q)\big) = 0$ or $\frac{dX^*(Q)}{dQ} = 0$. The first of these alternatives is just the *Smooth Pasting* condition Eq. (7) and implies incremental investment at the threshold $X^*(Q)$ captured by Eq. (8), whereas the second alternative implies a constant investment threshold function and a mass of investment. Specifically, in the range where the entry threshold is constant, the dynamic pattern described in figure 1 does not hold and an incremental investment does not raise the entry threshold rendering the market once again under the investment threshold. Instead, the threshold is unchanged and the market is still on it so investment continues, bringing market quantity immediately to $\bar{Q}$.

We therefore analyze the industry equilibrium with policy intervention in two steps, starting with the case of industry capacities $Q > \bar{Q}$, which means that the policy change has already taken place. We then address the case where the policy change has not yet occurred, $Q \leq \bar{Q}$.

For $Q > \bar{Q}$, no future policy changes are expected so the analysis is identical to the previous section with K' replacing K. The general form of the value of an active firm is therefore once again given by Eq. (3), and the Value Matching and Smooth Pasting conditions, given by Eq. (6) and (7), yield the entry threshold:

(12) $$X^{**}(Q) = \hat{\beta} \cdot (r - \mu) \cdot \frac{K' + \frac{w}{r}}{f(Q)}.$$

and also lead to

(13)
$$Y^{**}(Q) = -\frac{(\beta-1)^{\beta-1}}{\beta^{\beta}\cdot(r-\mu)^{\beta}\cdot\left(K'+\frac{w}{r}\right)^{\beta-1}}\cdot\left(f(Q)\right)^{\beta}$$

We now turn to the case where $Q \leq \bar{Q}$ so the policy change has not yet occurred. Since at this range the policy has not changed yet, the Value Matching condition for the optimal threshold is still given by Eq. (6), throughout this range.

We now focus on the right-end of this range where market quantity is $\bar{Q}$. Recalling that the term $Y(Q)$ in the value function (3) represents how subsequent entries affect value, implies that when the market quantity is $\bar{Q}$ those entries are based on the policy that will be applied when $Q > \bar{Q}$ and are given therefore by applying $\bar{Q}$ in $Y^{**}(Q)$ as captured by Eq. (13). This boundary condition implies that, Smooth Pasting does not hold at $\bar{Q}$, because if it did then Eq. (3). (6) and (7) would lead to $Y^{*}(\bar{Q})$, based on Eq. (10) and not to $Y^{**}(\bar{Q})$, which is based on Eq. (13). Thus, smooth pasting does not hold at $\bar{Q}$ and by continuity it does not hold at a sufficiently close vicinity to its left. Instead, $\frac{dX^{*}(Q)}{dQ} = 0$ holds in that vicinity. We denote the quantity at the left end of this range by $\tilde{Q}$ and the value of the entry threshold at this vicinity by $\bar{X}$. Figure 2 presents the resulting threshold function. The threshold $\bar{X}$ is found via applying (3), in the value matching condition (6), at $\bar{Q}$. This yields

(14)
$$Y(\bar{Q})\cdot\bar{X}^{\beta}+\frac{\bar{X}\cdot f(\bar{Q})}{r-\mu}-\frac{w}{r}-K=0$$

where in this case $Y(\bar{Q}) = Y^{**}(\bar{Q})$ as captured by (13). Note that K' enters this condition via $Y^{**}(\bar{Q})$.

Appendix B shows that Eq. (14) has a unique root in $\bar{X}$ within the range $\left(0, X^*(\bar{Q})\right)$. The industry capacity level $\tilde{Q}$ at which the run occurs is defined in turn by $\frac{dX^*(Q)}{dQ} = 0$ at the range of the run, and can be found via

(15) $$X^*\left(\tilde{Q}\right) = \bar{X}.$$

Figure 2 plots the entry dynamics.

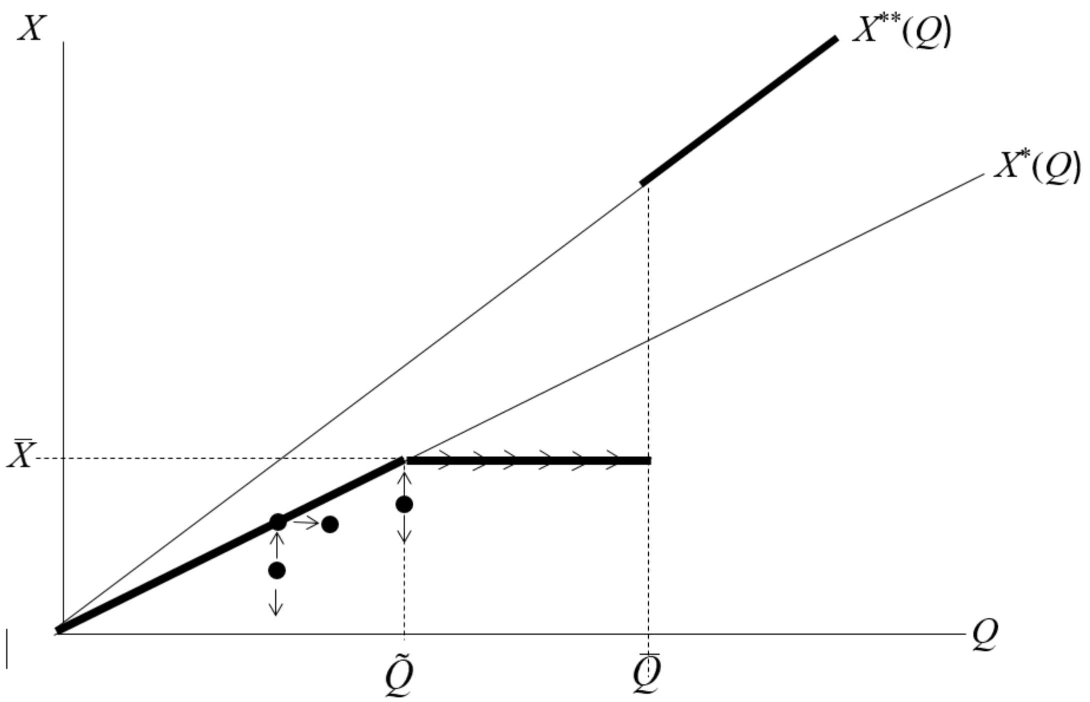

Figure 2. Entry threshold with an anticipated fixed cost increase at $\bar{Q}$

As Figure 2 shows, as long as $Q < \tilde{Q}$, entry follows a gradual process based on incremental increases in industry capacity at the points in time where $X^{*}(Q)$ is first hit. As soon as industry capacity hits $\tilde{Q}$ however, the anticipated change in fixed cost sparks a run and quantity instantaneously rises to $\bar{Q}$, maintaining firm value constant. Once this run has occurred, a gradual entry process resumes which is based on a higher threshold function $X^{**}(Q)$ corresponding to the new cost parameter value.

The second phase of gradual investment is a key difference with the case of a cap on industry capacity where investment ceases after $\bar{Q}$. Because of it, firms entering before the policy shock must anticipate an effect of further entries on their value $Y^{**}(Q)$ which is nonzero. This leads to the following sensitivity result.

Proposition 1. *The size of the run is increasing in the magnitude $K' - K$ of the policy intervention.*

Proof: Defining the RHS of (16) by $G[\bar{X}, Y^{*}(\bar{Q})]$ and differentiating, yields:

(16) $$\frac{d\bar{X}}{dY^{**}(\bar{Q})} = -\frac{\bar{X}^{\beta}}{Y^{**}(\bar{Q}) \cdot \beta \cdot \bar{X}^{\beta-1} + \frac{f(\bar{Q})}{r-\mu}}$$

$$< -\frac{\bar{X}^{\beta+1}}{Y^{**}(\bar{Q}) \cdot \bar{X}^{\beta} + \frac{\bar{X} \cdot f(\bar{Q})}{r-\mu}} = -\frac{\bar{X}^{\beta+1}}{\frac{w}{r} + K} < 0$$

where the inequality follows from $Y^{**}(Q) < 0$ taken together with $\beta > 1$. This , together with the positive effect of *K*' on $Y^{**}(\bar{Q}\ )$, as captured by (13), implies that the higher *K*' the lower $\bar{X}$ and therefore, by Eq. (15) and (8) the lower $\tilde{Q}$ too, which proves the proposition. □

The logic behind this result is as follows: ceteris paribus, a larger *K*' implies that the threshold policy after the policy change is going to be higher and, therefore, that future entries are going to be less frequent implying that the profit process is going to be better. To potential above normal profits attract firm to enter earlier, lowering thus both $\bar{X}$ and the value of $\tilde{Q}$, and making the range of the run wider.

It should be noted that the analysis in this section is not limited to the case where the increase in cost pertains to the one-time entry cost *K*, but also to the case where it ie levied on the flow of operating cost *w*. More specifically, equations (11)-(16) would be just the same, only with *w*' replacing *w* and *K* replacing *K*'.

5. Industry equilibrium with policy that affects all firms

In this section we study an alternative policy intervention. As in the preceding section, the intervention is triggered by an industry capacity $\bar{Q}$, and known to firms in advance. The effect of the policy intervention is to shift operating cost to $w' > w$, and in the current section we analyze a case where the cost increase applies to all firms, including those which were active before the change.

We analyze the industry equilibrium with policy intervention in two steps as in the preceding section. For $Q > \bar{Q}$, no future policy changes are expected so the analysis is identical to Section 2 with w' replacing w, which yields an expression for firm value

(17) $$V(Q,X) = Y(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w'}{r}$$

Applying Eq. (17) in the value Matching condition (6) and the Smooth pasting condition (7) yields the optimal entry threshold

(18) $$X^{**}(Q) = \hat{\beta} \cdot (r-\mu) \cdot \frac{K + \frac{w'}{r}}{f(Q)}.$$

And also yields that $Y(Q)$ is captured by

(19) $$Y^{(w')}(Q) = \frac{(\beta-1)^{\beta-1}}{\beta^{\beta} \cdot (r-\mu)^{\beta} \cdot \left(K + \frac{w'}{r}\right)^{\beta-1}} \cdot \left(f(Q)\right)^{\beta}$$

We now turn to the range $Q \leq \bar{Q}$. In that range, the general form of the value function is once again given by Eq. (3), with the term $Y(Q) \cdot X^{\beta}$ representing the magnitude by which future entries lower the value of the firm. This magnitude has two parts:

- First – future entries truncate the price process;
- Second – as the future entries take the market to $Q > \bar{Q}$ they raise the operating cost from *w* to *w*'.

This insight leads to the following boundary condition

(20) $$Y(\bar{Q}) \cdot X^{\beta} = Y^{(w')}(\bar{Q}) \cdot X^{\beta} - \frac{w' - w}{r} \cdot \left[\frac{X}{X^{**}(Q)}\right]^{\beta}$$

This condition is based on the case where the market quantity is exactly $\bar{Q}$. In that case, the first part of the two mentioned above, is based on future entries which take place under an operating cost of *w*'. Therefore, their effect on the value of the firm via the truncation of the price process is exactly $Y^{(w')}(\bar{Q}) \cdot X^{\beta}$, where $Y^{(w')}(\bar{Q})$ is captured by (19) and negative.

The second of the two parts by which future entries lower firm value is the increase of the operating cost once market quantity exceeds $\bar{Q}$. In the case where the market quantity is exactly $\bar{Q}$ and given the level of *X*, the value of this effect is given by the first term in (20). Specifically, it springs from the following calculation:

(21) $$E\left[\int_{T}^{\infty} (w' - w) \cdot e^{-r \cdot t} \cdot dt\right] = \frac{w' - w}{r} \cdot E(e^{-r \cdot T}) = \frac{w' - w}{r} \cdot \left[\frac{X}{X^{**}(Q)}\right]^{\beta},$$

where *T* is the first time when the threshold for the next entry, $X^{**}(\bar{Q})$ is hit, and the second equality is based on the formula for $E(e^{-r \cdot T})$ in Dixit and Pindyck (1994, page 315).

Applying Eq. (18) in (19) and then in Eq. (20) and rearranging terms, yields

(22) $$Y(\bar{Q}) = -\frac{X^{**}(Q) \cdot f(Q)}{\beta \cdot (r - \mu) \cdot [X^{**}(Q)]^{\beta}} - \frac{w' - w}{r \cdot [X^{**}(Q)]^{\beta}}$$

Applying Eq. (18) in Eq. (22) and simplifying further, yields:

$$Y(\bar{Q}) = -\frac{r \cdot K + \beta \cdot w' - (\beta - 1) \cdot w}{(\beta - 1) \cdot r \cdot [X^{**}(Q)]^{\beta}} \tag{23}$$

This boundary condition asserts that Smooth pasting does not hold at $\bar{Q}$. If it did, then Eq. (3), (6) and (7) would lead to the expression for $Y(\bar{Q})$ which is captured by Eq. (10), and differs from what condition (23) shows.[3]

By continuity, since smooth pasting does not hold at $\bar{Q}$ then it also does not hold at a sufficiently close vicinity to its left, with $\tilde{Q}$ denoting its left end. At this range the optimality condition for entry is provided by $\frac{dX^*(Q)}{dQ} = 0$ implying that throughout this range the entry threshold, once again denoted by $\bar{X}$, is constant. As in the previous section, $\bar{X}$ is the single root that Eq. (14) has within the range $\left(0, X^*(\bar{Q})\right)$, but with the difference that now the term $Y(\bar{Q})$ is given by Eq. (25) and not by Eq. (15). This difference does not alter the proof for the uniqueness of the solution within that range. Also similar to the analysis in the previous case is finding the industry capacity level $\tilde{Q}$ at which the run occurs via Eq. (15).

Proposition 2. In the case where the increase in cost applies to all firms, the size of the entry during the run, $\bar{Q} - \tilde{Q}$, is smaller than in the case where the increase in cost applies only to new entrants.

Proof: Implicit differentiation of Eq. (16) yields

[3] To verify this difference note that as *w*' converges to *w*, the RHS of (22) converges to that of (10), and that a straightforward differentiation of (22) yields $\frac{dY(\bar{Q})}{dw\prime} = \frac{\widehat{\beta}^2 \cdot (r-\mu) \cdot (\beta-1) \cdot (w'-w)}{r^2 \cdot f(\bar{Q}) \cdot [X^{**}(\bar{Q})]^{\beta+1}} > 0$

$$\frac{d\bar{X}}{dY^*(\bar{Q}\ )} = -\frac{\bar{X}^{\beta}}{\beta \cdot Y^*(\bar{Q}\ ) \cdot \bar{X}^{\beta-1} + \frac{f(\bar{Q})}{r-\mu}} \tag{24}$$

$$< -\frac{\bar{X}^{\beta}}{Y^*(\bar{Q}\ ) \cdot \bar{X}^{\beta-1} + \frac{f(\bar{Q})}{r-\mu}} = -\frac{\bar{X}^{\beta}}{K + \frac{w}{r}} < 0$$

Where the inequality follows from replacing β with 1, bearing in mind that $Y^*(\bar{Q}\ ) < 0$, and the second equality springs from Eq. (14).

Note from Eq. (22) that $Y^*(\bar{Q}\ )$ is smaller in the case where the cost is raised for all firms than it is in the case where it is raised only for new entrants. This, together with Eq. (24), implies that $\bar{X}$ is higher when the cost is raised for all firms, and therefore, by Eq. (15) and (8), $\tilde{Q}$ is larger (and $\bar{Q} - \tilde{Q}$ is therefore smaller) in the case where the change applies to all firms.

6. Welfare analysis

To study the normative consequences of competitive runs, we derive a welfare measure in this section and show how the run lowers welfare. To do this in the simplest way, we take the case of the policy intervention studied in Section 4 where a tax is imposed on fixed cost once industry capacity reaches a threshold level $\bar{Q}$ and also set operating cost to zero. The social cost of investment is taken to be K throughout. The tax, which amounts to $K' - K$, represents just a transfer from firms to the government. We also simplify by assuming that the social discount rate is r as well.

Tractability of the analysis requires using a particular form of the demand function, rather then the general one we have used so far. Specifically, we take the following constant elasticity demand with $\gamma < 1/\beta$, which implies that social welfare converges:

(25) $$P_t = X_t \cdot Q_t^{-\gamma}$$

We start by looking at the range $Q > \bar{Q}$ and defining the net social welfare in that range by $W^{**}(Q,X)$. A no-arbitrage argument, similar to the one conducted in Section 2 for the value of the firm, establishes that $W^{**}(Q,X)$ is a continuous and differentiable function of X, with the following general form,

(26) $$W^{**}(Q,X) = Z^{**}(Q) \cdot X^{\beta} + \frac{X}{r-\mu} \cdot \frac{Q^{1-\gamma}}{1-\gamma},$$

where $Z(Q)$ is to be found later via boundary conditions. Note that the first set of terms in Eq. (26) represents the effect of future entries on welfare, as the second is the perpetual welfare flow if industry capacity remains at its current level. To find $Z(Q)$ we follow Dixit and Pindyck (1994, page 286) and use the following condition for no arbitrage at time instant in which a new firm enters the market:

(27) $$W^{**}{}_Q[Q, X^{**}(Q)] = K.$$

Applying Eq. (25), (18), and (26) in Eq. (27), and simplifying, yields:

$$Z^{**\prime}(Q) = \frac{K - \hat{\beta} \cdot K'}{\hat{\beta}^{\beta} \cdot (r - \mu)^{\beta} \cdot K'^{\beta}} \cdot Q^{-\gamma \cdot \beta}. \tag{28}$$

Integrating Eq. (28), applying Eq. (18), and rearranging, yields:

$$Z^{**}(Q) = \frac{\hat{\beta} \cdot K' - K}{(\gamma \cdot \beta - 1) \cdot [X^{**}(Q)]^{\beta}} \cdot Q + C^{**} \tag{29}$$

The constant of integration takes the value $C^{**} = 0$, because as capacity becomes arbitrarily large no further entries occur, implying $\lim_{Q \to \infty} Z^{**}(Q) = 0$. Applying Eq. (29) in Eq. (26), yields that in the range $Q > \bar{Q}$ welfare is captured by

$$W^{**}(Q, X) = \frac{\hat{\beta} \cdot K' - K}{\gamma \cdot \beta - 1} \cdot Q \cdot \left[\frac{X}{X^{**}(Q)}\right]^{\beta} + \frac{X}{r - \mu} \cdot \frac{Q^{1-\gamma}}{1 - \gamma}. \tag{30}$$

In the range $Q \leq \bar{Q}$ we denote the social welfare function by $W^{*}(Q, X)$ and a similar analysis to the one carried for the complementary range leads to the general form:

$$W^{*}(Q, X) = \left[\frac{K \cdot Q}{(\gamma \cdot \beta - 1) \cdot (\beta - 1)} + C^{*}\right] \cdot \left[\frac{X}{X^{*}(Q)}\right]^{\beta} + \frac{X}{r - \mu} \cdot \frac{Q^{1-\gamma}}{1 - \gamma}. \tag{31}$$

where the integration constant, C^{*} is found via the following boundary condition:

(32) $$W^*(\tilde{Q},\bar{X}) = W^{**}(\bar{Q},\bar{X}),$$

which is based on the result that when the quantity is $\tilde{Q}$ and X hits the value $\bar{X}$, captured by Eq. (14), then with probability 1 a run occurs and quantity immediately rises from $\tilde{Q}$ to $\bar{Q}$.

Finally, we use a numerical example in this section to illustrate the effect of a run on industry investment and welfare. For this illustration we take standard parameter values, $r = 0.025$, $\mu = 0$, $\sigma = 0.1$, and $\gamma = 0.5$. We consider a small and large tax, i.e. one that raises fixed cost from $K = 250$ to $K' = 262.5$ and another that raises fixed cost to $K' = 500$. We set the capacity threshold for the tax at $\bar{Q} = 100$.

	$\bar{X}$	$\tilde{Q}$	$X^{**}(\bar{Q})$	Size of run ($\bar{Q} - \tilde{Q}$)	Welfare	Welfare benchmark
Small tax (5%)	71	54	100	46	37,674	59,907
Large tax (50%)	65	44	195	56	30,009	49,707

Table 1: the effect of the run on market dynamics and welfare.

In this example, the magnitude of the run is significant relative to the capacity $\bar{Q}$ which triggers it both with a small tax and with a large tax. Specifically, while the policy change occurs at $\bar{Q} = 100$, the run towards it starts already at $\tilde{Q} = 54$ under a 5% tax on the entry cost. This run towards $\bar{Q} = 100$ occurs when X hits the level $\bar{X} = 71$, while without the run the quantity $\bar{Q} = 100$ would have been reached only when X hits $X^{**}(\bar{Q}) = 95.2$. When the tax is raised by 50% the run occurs even earlier, at $\tilde{Q} = 44$ and $\bar{X} = 65$.

To see the welfare loss from policy anticipation we compare the actual welfare, as captured by Eq. (30) and (31), with a benchmark welfare based on an unanticipated policy change. In the latter case, where firms do not anticipate the policy change the industry investment follows the threshold policy $X^*(Q)$ up until $\bar{Q}$ and the updated threshold policy $X^{**}(Q)$ thereafter. Note that in the case of an unanticipated change welfare in the range $Q \leq \bar{Q}$ is still given by Eq. (31), but the integration constant, C^* is not found via the following boundary condition (32). Instead, the relevant boundary condition is:

$$W^*[\bar{Q}, X^*(\bar{Q})\,] = W^{**}[\bar{Q}, X^*(\bar{Q})], \tag{33}$$

Note that both in the benchmark and anticipated policy scenarios, the path of investment is the same up until $X^*(\tilde{Q})$ is first reached and the run occurs, as well as after $X^*(\bar{Q})$ is first reached so capacity in the benchmark scenario has caught up with the run. The welfare effect of a run is therefore due to the divergence in the two industry paths over the interval of time where industry capacity lies in $(\tilde{Q}, \bar{Q})$.

Based on that, to compare welfare with the run and with the unanticipated policy benchmark we measure welfare at the onset of the run, that is at the demand state $\bar{X}$ and with an industry capacity $\tilde{Q}$. As Table 1 shows, there is a 38% loss in welfare due to the run in the cases of a small tax, and a and a 40% loss in welfare in the case of a large tax. It should be noted though that the tax we consider is entirely distortionary and the threshold at which we measure it, $\bar{X}$, is also the threshold at which this difference is the largest. In the case of the smaller tax moreover, the percentage welfare loss is significantly greater than the tax itself.

7. Conclusion

In this paper, we have studied how the anticipation of a tax affects the equilibrium path of investment in a competitive industry and shown that, as in the case of a quota which had been the focus of the literature previously, a foreseeable policy change leads to a competitive run. Such a run causes a mass of firms to rush to take advantage of a transitory profitability increase ahead of the implementation of the tax, which is due to the less intensive entry process once the tax is in place. This phenomenon does not result from a coordination failure, and it occurs for a range of tax measures, whether these affect fixed or operating cost for example, and more generally so long as the policy change generates an upward jump in investment threshold function of inactive firms. Moreover, a straightforward numerical illustration using common parameters show that the magnitude of the run can be significant and the resulting loss of welfare can exceed the tax, in percentage terms.

Our results run counter to a conventional economic wisdom that announcing policies clearly and in advance is generally beneficial to economics actors, by highlighting a possible drawback that policy makers should bear in mind when economic actors have too precise a knowledge of policy timing. Our results also complement previous work on policy uncertainty which has generally emphasized that greater policy risk leads to investment delay, by showing that, at the other end of the spectrum, precise knowledge of a policy change results in a massive acceleration of investment. Finally, we have highlighted several factors which affect the magnitude of a run, such as the magnitude of the tax and whether the policy affects new entrants asymmetrically, which can help identify when these effects are liable to be germane to the analysis of policy measures, such as the subsidy withdrawal applications referred to in the introduction.

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Appendix A: The value of an active firm

In this Appendix we show that Eq. (3) represents the general form of the function $V(Q,X)$. For that, we use the standard no-arbitrage analysis of the literature on investment under uncertainty (see e.g. Dixit 1989). We start this analysis with the no-arbitrage condition,

$$r \cdot V(Q,X) \cdot dt = (X \cdot f(Q) - w) \cdot dt + E[dV(Q,X)], \tag{A.1}$$

which states that the instantaneous profit, $(X \cdot f(Q) - w) \cdot dt$, along with the expected instantaneous capital gain $E[dV(Q,X)]$ which springs from a change in X, must equal the instantaneous normal return, $r \cdot V(Q,X) \cdot dt$.

By Itô's lemma,

$$\frac{E[dV(Q,X)]}{dt} = \frac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(Q,X) + \mu \cdot X \cdot V_X(Q,X). \tag{A.2}$$

Substituting (A.2) in (A.1) yields:

$$\frac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(Q,X) + \mu \cdot X \cdot V_X(Q,X) - r \cdot V(Q,X) + X \cdot f(Q) - w = 0. \tag{A.3}$$

Trying a solution of the type X^b for the homogenous part of (A.3) and a linear form as a particular solution to the entire equation yields

(A.4) $$V(Q,X) = Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r} = 0$$

Where $Z(Q)$ and $Y(Q)$ are to be found later via additional conditions, and $\alpha < 0$ and $\beta > 1$ solve the quadratic

(A.5) $$\frac{1}{2} \cdot \sigma^2 \cdot x \cdot (x-1) + \mu \cdot x - r = 0.$$

Applying $x = 0$ and then $x = 1$, and bearing in mind that $r > \mu$ asserts that (A.5) has two roots, one of them negative and the other exceeds 1.

By the standard properties of a geometric Brownian Motions, it follows that[4]:

(A.6) $$E_{X_0=X}\left[\int_0^{\infty} X_t \cdot e^{-r \cdot t} dt\right] = \frac{X}{r-\mu}.$$

Eq. (A.6) implies that the term $\frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r}$ in (A.4) represents the expected value of the flow of profits if Q remains at its current level forever. The two other terms in (A.4) therefore represent how expected future changes in Q affect the value of the firm.

[4] Dixit and Pindyck (1994) provide a proof for (A.6). See the second example on page 82.

As a geometric Brownian motion, when X goes to 0 the probability of hitting $X^*(Q) > 0$, and thus of an increase in Q, tends to zero. Therefore

(A.7) $$\lim_{X \to 0} \left(Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta} \right) = 0.$$

Because $\alpha < 0$, (A.7) implies $Z(Q) = 0$. Substituting into (A.4) then gives Eq. (6) in the text.

Appendix B: Run threshold (fixed cost increase)

Defining the RHS of Eq. (16) as $G(\bar{Q}\ )$ leads to

(a) $G(0) = -\frac{w}{r} - K < 0$

(b) $G[X^*(\bar{Q})] > 0$

(c) $G"(\bar{Q}\ ) < 0.$

Which immediately asserts that Eq. 16 has a unique root in the range $\bar{X} \in \left(0, X^*(\bar{Q})\right)$. Note that (b) follows from

(B.1) $$G[X^*(\bar{Q})] = Y^{**}(\bar{Q}\ ) \cdot [X^*(\bar{Q})]^{\beta} + \frac{[X^*(\bar{Q})] \cdot f(\bar{Q})}{r - \mu} - \frac{w}{r} - K$$

$$> Y^*(\bar{Q}\ ) \cdot [X^*(\bar{Q})]^{\beta} + \frac{[X^*(\bar{Q})] \cdot f(\bar{Q})}{r - \mu} - \frac{w}{r} - K = 0$$

where the inequality follows from (10), (13) and K ' > K, and the second equality follows from the Value Matching condition (6), taken together with (3) and (10). (c) follows from

(B.2) $$G''(\bar{X}) = Y^{(K')}(\bar{Q}\ ) \cdot \beta \cdot (\beta - 1) \cdot (\bar{X})^{\beta-2} < 0$$

where the inequality follows from $Y^{**}(\bar{Q}\ ) < 0$ as captured by (13).